angles are expressed by integers, providing the three sides of each triangle be integers. But from (8) we find that the sides of the new triangles may have a common factor. Guarding this point, we may form from a table of the lowest integers representing sides of a right triangle a corresponding table for a scalene triangle whose *area is an integer*. A four-page table of this kind is given in Dr. Halsted's *Mensuration*.

POSTULATE I. OF EUCLID'S ELEMENTS.

By Professor JOHN L. LYLE, Ph, D., Westminister College, Fulton, Missouri.

"Let it be granted that a straight line may be drawn from any one point to any other point." Euclid lays down the statement just quoted as his first postulate regulative of geometrical constructions. Wherever any two points may be located in unbounded space, Euclid assumes that a straight line may be drawn from one of them to the other.

Such a line is finite in length, of course, according to the definition that "a *finite* straight line is one that has *two ends*".

The assumption that a straight line of *infinite* (boundless) length can be drawn between two points in space is not only anti-Euclidean but also destructive of the logical law of non-contradiction which forbids that contradictory marks shall be attributed to any straight line whatever. A line can not have two ends and at the same time be without ends,(infinite,that is,unbounded).

In every rectilineal triangle, there are three angular points and each of the three sides may be constructed in strict harmony with the 1st postulate of Euclid. Each of these sides has the distinctive marks of a finite straight line, to wit: two ends.

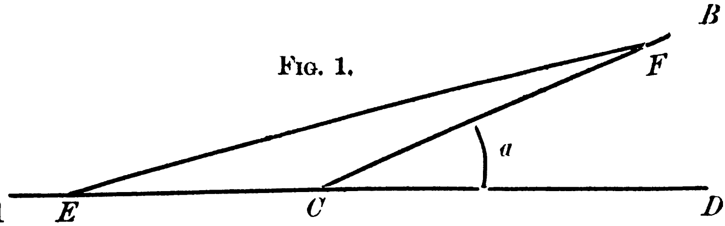
Euclid proves in proposition XVII, Book I., that "Any two angles of a triangle are together less than two right angles". This proposition is the converse of Euclid's 12th axiom about which so much has been written.

John Playfair states that axiom as follows: "If a straight line meet two straight lines, so as to make the interior angles on the same side of it less than two right angles, these straight lines being continually produced will at length meet on the side on which the angles are less than two right angles."

Lobatschewsky, in his theorem 19,demonstrates the proposition that the angle-sum of a rectilineal triangle can not be greater than two right angles. Then assuming the falsity of Euclid's 12th axiom, he concludes (whether logically or illogically) that the angle-sum of a rectilineal triangle is less than two right angles.

But a single angle ACB can always be drawn less than two right angles and in such manner as that it shall differ from two right angles by as small an angle a as we please. The single angle ACB may, therefore, be made equal to any assumed sum whatever (less than two right angles) of the three angles of a recti-

Fig. 1.



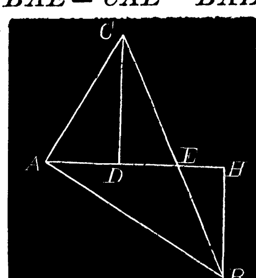
lineal triangle. Let the angle ACB, Fig. 1, be equal to the assumed angle-sum of a rectilineal triangle, to wit: two right angles$-a$. Then, a straight line can not be drawn from any point E on the leg AC to any point F on the leg CB; for if that is granted, we shall have a triangle ECF whose angle-sum is greater than two right angles$-a$, which is against the hypothesis. But to refuse to grant that a straight line may be drawn from any point E to any other point F is to discredit postulate 1, of Euclid's Elements. The assumptions of Lobatschewsky's geometry are at war with Euclid's 1st postulate.

DEMONSTRATION OF THE "TANGENT PROPOSITION," AND OTHERS.

By Professor P. H. PHILBRICK, M. S., C. E., Lake Charles, Louisiana.

Let ABC be the triangle considered. Represent the parts as usual. Make $CE = CA$ and draw CD and BH perpendicular to AE and AE prolonged.

We have $A = CAE + BAE$; and $B = AEC - BAE = CAE - BAE$. Adding and dividing by two, gives $CAE = \frac{1}{2}(A+B)$. Subtracting and dividing by two gives $BAE = \frac{1}{2}(A-B)$.



The triangles ACD, ECD, and BEH are similar.

Hence, $\dfrac{AD}{EH} = \dfrac{AC}{BE}$, $\dfrac{DE}{EH} = \dfrac{CE}{BE}$, and $\dfrac{EH}{EH} = \dfrac{BE}{BE}$.

Adding, we have $\dfrac{AD+DE+EH}{EH} = \dfrac{AC+CE+EB}{BE}$.

That is, $\dfrac{AH}{EH} = \dfrac{a+b}{a-b}$ or $\dfrac{EH}{AH} = \dfrac{a-b}{a+b}$.

Also, $\dfrac{BH}{AH} = \tan\frac{1}{2}(A-B)$ and $\dfrac{BH}{EH} = \tan\frac{1}{2}(A+B)$.